A Proposal of a Set of Metrics for Collective Movement of Robots

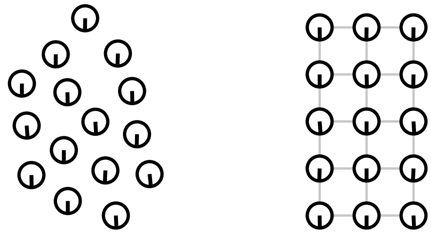
Iñaki Navarro and Fernando Matía
Intelligent Control Group
Universidad Politécnica de Madrid
José Gutierrez Abascal, 2, 28006 Madrid, Spain
Email: {inaki.navarro,fernado.matia}@upm.es

***Abstract*— In this paper a set of metrics that measures the performance of collective movement of mobile robots is proposed and discussed. Coordination of movement of robot groups represents a basic problem in multi robotics: individuals must move together as a group maintaining certain relative positions between them. The different metrics proposed here cover several aspects of the characteristics of the collective movement including: those related to the area and shape of the group, the movement, the positioning and orientation of its members, and the efficiency and success of the algorithms. Some of them can be used as a benchmark, while others might be useful to characterize the algorithms in terms of group behavior.**

I. Introduction

There are many robotic applications in which a multi robot approach might be an advantage compared to single robot systems, showing desirable properties like robustness, flexibility and scalability [1]. Coordination of movement of groups of robots represents a basic problem in multi robotics. This problem is called formation of mobile robots or flocking of robots. In flocking of robots (e.g. [2, 3, 4]) individuals move as a group but the shape and relative positions between the robots are not fixed, allowing robots to move within the group. Then again, formations of mobile robots are more strict regarding the positions that the robots must occupy (e.g. [5, 6, 7]). Members must maintain pre-determined positions and orientations among them at the same time that they move as a whole. Having a pre-defined global shape is not always a requirement, and sometimes only relative positions between the members are defined. Examples of flocking and formations of robots are shown in Fig. 1.

In this paper we intend to define and discuss a set of metrics to evaluate the performance of collective movement of mobile robots. As stated in [8], the set of measurements used in robotic experiments to define its performance must be identified and clearly motivated when reporting results. In order to compare different algorithms, it is necessary to use common metrics for the different algorithms. This set of metrics should go with a set of scenarios and tests, similar to those described in [9], where the experiments should be carried out, in order to have a benchmark. Anyway, this work concentrates on the metrics.



(a) (b)

Fig. 1. (a) A schematic of a flock of 15 robots. (b) A schematic of a formation of 15 robots.

The multi robot research community working on formations and flocking of robots do not share a common set of metrics, and usually similar but not comparable metrics are used. There are few works on defining metrics for collective movement, like [10] and [11], but the first does not cover some of the metrics that can measure collective movement, while the second is focused on collective movement of fish schools. This is a reason to propose a more complete set of metrics oriented to analyse and discuss experiments on formations and flocking of robots. In principle the metrics are focused on 2 dimensions, but could be extrapolated to 3-dimensional problems with formations of aquatic robots or autonomous flying vehicles.

The basic mathematical notation used in the paper is defined in Section II. In Section III the methodology for acquiring the required data is pointed out. The set of proposed metrics is defined and discussed in Section IV. Finally an overview of the paper and future work can be found in Section V.

II. Basic Definitions

We can define the complete group of robots under study as $R = \{..., r_i, ...\}, 1 \leq i \leq N$, where N is the number of robots. The position at time t of a single robot r_i can be defined with respect to an arbitrary reference frame by the pose vector $\vec{\xi}_i(t) = (x_i(t), y_i(t), \theta_i(t))^T$, where $x_i(t)$ and $y_i(t)$ are the coordinates of a reference point $\vec{p}_i$ of the robot r_i, and θ_i describes the orientation of a frame attached to the robot with respect to the reference frame, this is, its heading. The positions and heading of the group of robots R is then given by $\vec{\xi}(t) = (..., \vec{\xi}_i^T(t), ...)^T, 1 \leq i \leq N$.

The δ-neighborhood of a robot is defined as a set of robots that fulfill certain conditions based on inter-robot distance, that depends on the parameter δ. The δ-neighborhood $\mathcal{N}_{i,\delta}(t)$ of a robot r_i is $\mathcal{N}_{i,\delta}(t) = \{r_j | j \neq i, \|\vec{p}_i(t) - \vec{p}_j(t)\| \leq \delta\}$, where

$\delta > 0$ and it is set arbitrarily depending on the experiment under analysis.

The robots moving in formation or flocking communicate and detect the relative positions of their neighbors under a certain range, building a graph that determines the connectivity of the group. In [12] the notion of *dynamic undirected graph* is described and some of its properties are explained, which will be here summarized. A *dynamic graph* is defined as $\mathcal{G}(t) = (R, \mathcal{E}(t))$ where $R = \{..., r_i, ...\}, 1 \leq i \leq N$ is the set of vertices or robots and $\mathcal{E}(t) = \{(r_i, r_j)|r_i, r_j \in R\}$ denotes the time varying set of links such that: $(r_i, r_j) \in \mathcal{E}(t)$ if $\|\vec{p}_i(t) - \vec{p}_j(t)\| \leq range$ and $(r_i, r_j) \notin \mathcal{E}(t)$ if $\|\vec{p}_i(t) - \vec{p}_j(t)\| > range$, where $range$ is the range of the communication and neighboring detection system. Since the communication is assumed to be bidirectional and $\|\vec{p}_i(t) - \vec{p}_j(t)\| = \|\vec{p}_j(t) - \vec{p}_i(t)\|$, then $(r_i, r_j) \in \mathcal{E}(t)$ if and only if $(r_j, r_i) \in \mathcal{E}(t)$, so the graphs under study are undirected. According to the previous definition of $\mathcal{N}_{i,\delta}$ it can be pointed out that $(r_i, r_j) \in \mathcal{E}(t)$ if and only if $j \in \mathcal{N}_{i,\delta}$ for a $\delta = range$. If in $\mathcal{G}(t)$ there is a path (i.e., a sequence of distinct vertices such that consecutive vertices are adjacent) between any two of its vertices, then it can be said that $\mathcal{G}(t)$ is *connected*.

In order to compute the different proposed metrics, a sampled version of the vectors and equations will be used, since both data acquisition and computation are discrete. The sample period used is T, while n indicates the sample number that corresponds to $t = nT$ and M is the number of samples of a complete experiment. For instance the pose vector $\vec{\xi}_i(t)$ of robot r_i is expressed in its sampled form as: $\vec{\xi}_i[n] = \vec{\xi}_i(nT) = (x_i(nT), y_i(nT), \theta_i(nT))^T = (x_i[n], y_i[n], \theta_i[n])^T$.

III. Methodology

In order to apply the proposed metrics, the pose for all the robots $\vec{\xi}(t)$ together with the time must be logged with a certain frequency ($1/T$). These data can be easily acquired in simulation since pose information is directly available, but it becomes more complicated when working with real robots. The simplest solution is to use an external system, usually an overhead camera aiming at the floor together with a tracking system (e.g. [13]), that logs the poses of every robot periodically for a subsequent analysis. There are situations where an external system can not be used and pose information must be retrieved directly from the robots. Robots used in collective robot movement posses systems capable of estimating the relative positions of nearby robots, using infra-red [14] or vision systems [15]. Relative positioning information, combined with odometry for all the robots, might be processed off-line in order to estimate the pose of each robot at a given time step.

IV. Metrics

The proposed set of metrics seeks to be as complete as possible but also useful to determine the performance of the different algorithms for the coordination of collective movement. Some of the metrics can be applied only to formations of robots or to flocking of robots, but not to both problems due to the different nature of them. Not all the metrics proposed have the objective of being used as a benchmark. There are some that are useful to characterize the type of flock, but that can not be used to create a ranking.

The set of metrics has been divided in different subsets, grouping them according to their resemblances. They are defined and discussed in the following subsections.

A. Shape

In order to compute the metrics related to the shape of the group, the robots that belong to the boundary of the flock or formation must be identified. This can be done using convex hull [16] or α-shape [17] algorithms, leading to different results in the metrics.

The convex hull of a set of points S, is the smallest convex polygon containing all the points in S, and it is defined by the sub-set of points in S that are vertices of the convex polygon. The algorithm described in [16] computes the convex hull given a set of N points in the plane with time complexity O(N log N).

The α-shape algorithm represents a generalization of the convex-hull. It allows to calculate the α-hull to determine a more generalised boundary of a set of points, that depends on the arbitrary α parameter. As stated in [17], a *generalized disk* of radius $1/\alpha$ is defined as an ordinary closed disk of radius $1/\alpha$ if $\alpha > 0$; a half-plane if $\alpha = 0$; and the complement of a closed disk of radius $-1/\alpha$ if $\alpha < 0$. Given a set of points S the α-hull of S is the intersection of all closed *generalized disks* of radius $1/\alpha$ that contain all the points in S. Setting an appropriate negative value of α, non-convex polygons that better reflect the structure of the flock/formation can be obtained. The main disadvantage of the α-shape algorithm is that it is dependant on the α parameter which must be set in relation to the desired inter-robot distance. In Fig. 2 two examples of boundaries calculated with the convex hull and α-shapes are shown.

Once the set of robots that belong to the boundary $R_{boundary}[n]$ is identified, both the area $A_R[n]$ and the perimeter $P_R[n]$ can be computed for every n sample, and more complicated metrics may be calculated from them. The robots belonging to the boundary $R_{boundary}[n] = \{..., rb_k[n], ...\}, 1 \leq k \leq L[n]$, where $L[n] = |R_{boundary}[n]|$ is the number of elements in this set.

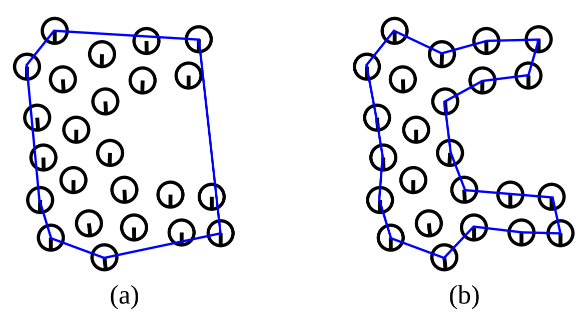


Fig. 2. The boundary of a group of robots is identified. (a) Using convex hull. (b) Using α-shapes.

1) Area: The area that encloses the group of robots, $A_R[n]$, can be used to identify if the area of the group grows too much, which might indicate that the flock or formation is being split in small groups. But this metric depends on the number of robots N and on the desired inter-robot distance of both flocking and formations. If the desired inter-robot distance is not changed over the experiment and $A_R[n]$ does not increase above a certain threshold after an initialization time, it will be an indicator that the robots remain together.

2) Robot Density: It represents the number of robots per m^2, and it is defined as:

$$RD_R[n] = \frac{N}{A_R[n]} \tag{1}$$

As $A_R[n]$, it can be used to determine if the group remains together, and the advantage is that $RD_R[n]$ is not dependent on the number of robots (N).

3) Isoperimetric Quotient: It is defined as:

$$IQ_R[n] = \frac{4\pi A_R[n]}{P_R[n]^2} \tag{2}$$

It measures the ratio of the area enclosed by the curve to the area of a circle with the same perimeter [18]. It gives an idea of the evolution of the shape of the flock or formation. If all the robots in R form a straight line then $IQ_R[n] = 0$. The maximum value is obtained when boundary robots form a perfect circle, so $IQ_R[n] = 1$.

4) Mean Distance from the Boundary to the Center of the Group: It is the mean distance from every robot rb_k belonging the boundary $R_{Boundary}$ to the center of mass of the group. The center of mass of the group at sample n can be defined as:

$$\vec{C}_R[n] = \frac{1}{N}\sum_{i=1}^{N} \vec{p}_i[n] \tag{3}$$

The mean distance from the boundary to the center of the group is expressed as:

$$\vec{M}DBC_R[n] = \frac{1}{L[n]}\sum_{k=1}^{L[n]} \|\vec{p}_{bound-k}[n] - \vec{C}_R[n]\| \tag{4}$$

where $p_{bound-k}$ is the position of robot rb_k belonging to $R_{Boundary}$. This metric represents a measure of the spatial size of the group of robots.

The metrics in this section are in general useful to identify some properties and behaviors of the tested algorithms, but they cannot be used to compare quantitatively different experiments.

B. Movement

This subset pretends to characterize the movement and trajectory of the group of robots.

1) Group Velocity: It is the velocity of the center of mass of the group.

$$G\vec{V}_R[n] = \frac{\vec{C}_R[n] - \vec{C}_R[n-1]}{T} \tag{5}$$

2) Average Group Speed: It is the average speed over the whole experiment of the center of mass of the group.

$$AGS_R = \frac{1}{M}\sum_{n=1}^{M} \|V\vec{G}_R[n]\| \tag{6}$$

3) Length of the Trajectory: It is the length of the trajectory followed by the center of mass of the group.

$$LT_R = \sum_{n=2}^{M} (\|\vec{C}_R[n] - \vec{C}_R[n-1]\|) \tag{7}$$

$G\vec{V}_R[n]$ and more specifically its magnitude ($GS_R[n]$) can be plotted over the time to get an idea of the evolution of the experiment. AGS_R can be used to compare the quality of different algorithms, the higher average speed the better. LT_R it is used to calculate other metrics.

C. Quality

This subset of metrics measures the quality of the algorithms of flocking and formation of robots in terms of error in the positioning and the heading. Some of them can be only used in formations while others are recommended for certain flocking algorithms. They can be used to compare the different algorithms in terms of error positioning and error orientation of their members.

1) Mean Position Error: This metric is specific to formations of robots, where robots have pre-defined positions to occupy. These positions can be defined with respect to the local frame of a leader robot. We impose that robot r_1 is the leader robot. The desired positions of the neighboring robots referred to the leader frame are $\{..., \vec{p}_{des-i}, ...\}, 1 \leq i \leq N$, where $\vec{p}_{des-l} = (x_{des-l}, y_{des-l})$. These positions are translated to the global reference frame, $\vec{p}_{des-glob-i} = (x_{des-glob-i}, y_{des-glob-i})$ where:

$$x_{des-glob-i} = x_1 + x_{des-i}\cos(\theta_1) - y_{des-i}\sin(\theta_1) \tag{8}$$

$$y_{des-glob-i} = y_1 + x_{des-i}\sin(\theta_1) + y_{des-i}\cos(\theta_1) \tag{9}$$

Taking into account this definitions the mean position error (averaged over the different robots) can be defined as:

$$MPE_R[n] = \frac{1}{N}\sum_{i=1}^{N} \|\vec{p}_{des-glob-i}[n] - \vec{p}_i[n]\| \tag{10}$$

The desired pre-defined positions of the robots could also be defined with respect to a frame placed in the center of mass of the formation and with heading equal to the mean global heading of all the robots.

$MPE_R[n]$ can be averaged over the time duration of an experiment or several experiments to get a unique value and be able to compare how good different algorithms are with respect to the positioning of the robots in the formation. The average of $MPE_R[n]$ over time is:

$$AMPE_R = \frac{1}{M}\sum_{n=1}^{M} MPE_R[n] \tag{11}$$

2) Mean Distance Error: There exist certain types of formations in which robots form triangular lattice patterns, maintaining a certain inter-robot distance to their closest neighboring robots. Some flocking algorithms also create these triangular lattice. In these kind of formations and flocks the inter-robot distance error averaged over the different robots and neighbors can be calculated in order to get a value of how well positioned the robots are, instead of calculating $MPE_R[n]$.

In order to calculate the mean distance error MDE_R, neighbors at a $distance < 1.5 * desiredInterRobotDistance(dd)$ will be considered to compute the distance error. For a robot r_i its neighbors will be those belonging to $\mathcal{N}_{i,\delta}$ for a $\delta = 1.5{*}dd$.

$$MDE_R[n] = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{|\mathcal{N}_{i,\delta}[n]|}\sum_{r_j\in\mathcal{N}_{i,\delta}[n]}|(\|\vec{p}_j[n]-\vec{p}_i[n]\|)-dd|\right) \quad (12)$$

3) Robot Density Error: It computes the difference between the RD_R and an expected robot density $RD_{Desired}$. In essence it can be used in flocking, when the algorithm tries to maintain robots at a desired inter-robot distance, and the flock is not too loose in its positioning. It can be also used in robot formations. The $RD_{Desired}$ must be calculated for each experiment or algorithm depending on the desired inter-robot distance.

$$RDE_R[n] = RD_R[n] - RD_{Desired} \quad (13)$$

This metric is used, as $MDE_R[n]$, to show if robots maintain the desired inter-robot distance or expected robot density.

4) Average Density Error: It is the average over time of the experiment of the $RDE_R[n]$.

$$ARDE_R = \frac{1}{M}\sum_{n=1}^{M}|RDE_R[n]| \quad (14)$$

This value is useful to compare different algorithms. If algorithms with different $RD_{Desired}$ must be compared a normalized $NARDE_R$ value of $ARDE_R$ might be used:

$$NARDE_R = \frac{ARDE_R}{RD_{Desired}} \quad (15)$$

5) Mean Orientation Error: This metric aims to measure how well oriented the robots are among the group. The error with respect a reference orientation is averaged over the different robots. This reference orientation θ_{ref} can be the heading mean of all the robots or the desired direction movement of the flock or formation in case it is known by the system that calculates the metrics. The heading mean can be calculated as follows: $HM_R = arg(\sum_{i=1}^{N}\vec{u}_{\theta_i})$, where $\vec{u}_{\theta_i}$ is a vector of magnitude 1 and direction θ_i. The mean orientation error is:

$$MOE_R[n] = \frac{1}{N}\sum_{i=1}^{N}|\theta_i[n] - \theta_{ref}[n]| \quad (16)$$

If robot orientations are equally spaced then $\sum_{i=1}^{N}\vec{u}_{\theta_i} = \vec{0}$, so HM_R will be undetermined, but in this case $MOE_R[n] = \frac{\pi}{2}$, independently of θ_{ref}.

6) Polarization: This metric is similar to MOE_R and also aims to measure how well oriented the robots are. Polarization [19] Po_R of a group of robots R is defined using the angular nearest neighbor. For a robot r_i, the corresponding angular nearest neighbour r_{ann} is defined so that $\theta^{r_i r_{ann}}$, the relative orientation of r_{ann} with respect to r_i is as small as possible: $\theta^{r_i r_{ann}} = min(\theta^{r_i r_j}), \forall r_j \neq r_i \in R$. We denote $\theta_{ann}(r_i, \xi[n])$ the relative orientation of the angular nearest neighbour of the robot r_i. The formal definition of polarization is defined as follows:

$$Po_R[n] = \sum_{i=1}^{N}\theta_{ann}(r_i, \xi[n]) \quad (17)$$

If all robots are aligned, then $Po_R = 0$. Conversely, if headings are evenly distributed, $Po_R = 2\pi$. Lastly, if headings are random, *i.e.* drawn from a uniform distribution, then $Po_R = \pi$ in average. Meaningful comparisons among different group sizes are possible since the average value of Po_R is not affected by the number of robots in R.

7) Percentage of Time in Formation: This metric also represents a measure on how well positioned robots are in the formation during an experiment. We can define the function *Is in Formation* to determine if the group is in formation taking into account the MPE_R:

$$IIF(R)[n] = \begin{cases} 1 & \text{if} \quad MPE_R[n] < e \\ 0 & \text{if} \quad MPE_R[n] \geq e \end{cases} \quad (18)$$

where e is a certain arbitrary threshold. Then percentage of time in formation can be calculated as:

$$PTF_R = \frac{\sum_{n=1}^{M} IIF(R)[n]}{M} * 100 \quad (19)$$

8) Time to Be In Formation: This metric calculates the time it takes the group of robots to reach $MPE_R(t) < e$, where is a certain arbitrary error threshold, as in the case of PTF_R. It can be used to measure how long it takes an algorithm to create a formation, but also to calculate other metrics like AGS_R, average over time of Po_R, and $AMPE_R$, not from $t = 0$ but from time to be in formation $t = t_{IF}$.

D. Efficiency and Cost

The metrics of this subset try to measure the efficiency in terms of path followed by the robots and energy consumption. They are useful both to characterize the algorithms and to compare them.

1) Path Length Ratio: It is the ratio between path length followed by the center of mass of the group and straight line distance from the initial to the ending point.

$$PLR_R = \frac{LT_R}{\|\vec{C}_R[0] - \vec{C}_R[M]\|} \quad (20)$$

In case of absence of obstacles the interpretation is straight forward and the minimum value $PLR_R = 1$ means an optimal movement of the flock or formation. When obstacles are present it is necessary to compare between different algorithms or reformulate the metric to compute the ratio between the followed path length and the minimum possible path from starting point to ending point.

2) Single Robot Path Length Ratio: It is a similar metric to PLR_R, but instead of using the path length of the center of mass to calculate the ratio, the average of the path lengths of every robot is used.

$$SRPLR_R = \frac{\frac{1}{N}\sum_{i=1}^{N}\left(\sum_{n=2}^{M}(\|p_i[n]-p_i[n-1]\|)\right)}{\|\vec{C}_R[0]-\vec{C}_R[M]\|} \quad (21)$$

3) Ratio Energy Consumption Distance Travelled: It is the ratio between the average of the energy consumption EC_i in the experiment over the different robots and the straight line distance from the initial to the ending point.

$$RECDT_R = \frac{\frac{1}{N}\sum_{i=1}^{N} EC_i}{\|\vec{C}_R[0]-\vec{C}_R[M]\|} \quad (22)$$

In order to compute this metric it is also necessary to measure and report the energy that every robot uses during the experiment, which makes this metric more difficult to use.

E. Grouping

In collective movement of robots the group of robots R can split in several subgroups R_k, due to fails in the algorithm or on purpose in order to overcome an obstacle or just to divide and explore more efficiently the environment. The splitting in several subgroups must be detected in order to compute other metrics. For instance the trajectories followed by two split subgroups R_1 and R_2 must be calculated in a separated way using the centers of mass of of the different groups $\vec{C}_{R_1}$ and $\vec{C}_{R_2}$ and not the center of mass of the complete group $\vec{C}_R$.

The different subgroups $\{..., R_k[n], ...\}, 1 \leq k \leq K[n]$ can be calculated using the connectivity properties of of $\mathcal{G}[n]$, where $K[n]$ is the number of subgroups at sample n. If $\mathcal{G}[n]$ is is *connected* then $R_1[n] = R$ and there exists just one subgroup ($K[n] = 1$). Otherwise in order to determine the different subgroups R_k and their members r_i the *connected* sub-graphs of $\mathcal{G}[n]$ must be identified. A possible algorithm to do it is the following:

```
S = R
i = 0
WHILE S is not empty
  i = i+1
  rs = a robot in S
  Ri = Ri + {rs}
  S = S - {rs}
  FOR each robot raux in S
    IF there is path from rs to raux
      S = S - {raux}
      Ri = Ri + {raux}
    ENDIF
  ENDFOR
ENDWHILE
```

where `R` is the group of robots R, the operation `Ri = Ri + {rs}` adds the robot r_s to the subgroup R_i and `S = S - {rs}` removes the robot r_s from the set S.

1) Number of Subgroups: It is the number of subgroups in which the whole group R is split.

$$NS_R[n] = K[n] \quad (23)$$

2) Size of a Subgroup: It is the number of robots that belong to a certain subgroup R_k.

$$S_{R_k}[n] = |R_k| \quad (24)$$

If $S_{R_k}[n] = N$ then $R_k[n] = R$. If $S_{R_k}[n] = 1$, then the only robot belonging to the subgroup R_k is isolated.

F. Success

These metrics measure the success of the algorithm in terms of robot failures and number of robots arriving together to the end. It can be used to compare the robustness of different algorithms.

1) Ratio of Lost Robots: This metric is used to measure the number of robots that have lost the main subgroup. If we define the main subgroup R_m as the subgroup with maximum number of robots then:

$$RLR_R[n] = \frac{N - |R_m[n]|}{N} \quad (25)$$

An interesting value is the value at the end of the experiment $RLR_R[M]$.

2) Ratio of Not Working Robots: There are many ways to determine if a robot is not working any more. One of them could be to determine if the robot is not moving for a while, but also robots could identify themselves as not working to report afterwards their state to the system in charge of calculating this metric. If $NWR[n]$ is the number of not working robots at sample n, then:

$$RNWR_R[n] = \frac{NWR[n]}{N} \quad (26)$$

G. Security

The metric defined in this subset measures the security of the flock, in terms of distance to obstacles.

1) Percentage of Time Group of Robots is Too Close to Obstacles: Distance to obstacles can be recorded using the in-built sensors of robots and coupling it with the information from the overhead camera. It might also be retrieved directly from the tracking system by placing landmarks on top of the obstacles, in the case of very structured laboratory environments. We define the function $TC_R[n]$ whose result is 1 if the group is too close to any obstacle and 0 otherwise:

$$TC_R[n] = \begin{cases} 0 & \text{if} \quad \forall r_i \in R, \forall o_l, \|p_i - p_{o_l}\| > sd \\ 1 & \text{otherwise} \end{cases} \quad (27)$$

where p_{o_i} is the position of all the points in the environment that describe the boundaries of obstacles, and sd is the security distance that the group of robots must maintain with obstacles. Then,

$$PTTCO_R[n] = \frac{\sum_{n=1}^{M} TC_R[n]}{N} * 100 \quad (28)$$

H. Other

1) Mean Number of Neighboring Robots: This metric calculates the mean number of neighboring robots over the robots in R, showing how connected $\mathcal{G}[n]$ is.

$$MNNR_R[n] = \frac{1}{N} \sum_{i=1}^{N} |\mathcal{N}_{i,\delta}[n]| \tag{29}$$

I. Higher Order Comparisons

Different types of comparisons can be done using two or more of the above described metrics. For instance in order to measure the scalability of the algorithms with increasing number of robots, the variation of metrics like AGS_R, $AMPE_R$, and $ARDE_R$ can be measured and plotted for different values of N. If AGS_R does not decrease and the errors $AMPE_R$ and $ARDE_R$ do not increase with N increasing it will indicate that the algorithm is scalable.

In order to see how the error in positioning may affect the group speed and see if there exist a correlation between both metrics, values of $GS_R[n]$ and $RDE_R[n]$ for different time steps n can be plotted in a graph using GS_R and RDE_R as variables of the coordinate axes.

V. Overview and Further Work

Several metrics for collective movement of robots have been defined and briefly discussed. Some of them are useful just to characterize the algorithms while others can be used to compare their performance.

The use of real examples comparing two or more experiments could help to better understand some of the metrics. Moreover, the use of the metrics when the group of robots R is split in several subgroups R_i must be discussed in more detail. Also some of the metrics depend on arbitrary parameters which makes the comparison between different algorithms difficult.

As it has been previously stated, in order to be able to compare different algorithms, it is necessary not only to have a set of metrics but to share a common set of environments with obstacles and places that groups of robots must reach. This must be done both in simulation and in the real world, defining simple laboratory environments that can be easily reproduced. In addition, the use of a common platform would allow to better compare results, but in our opinion we are far from reaching that situation. The long-term aim is to build a more complete reference set of metrics, together with different experimental sets in order have benchmarks to compare collective movement algorithms.

Acknowledgment

Authors are partially sponsored by Spanish Ministry of Science (ROBONAUTA project DPI2007-66846-C02-01). I. Navarro is sponsored by Madrid Region with a Ph.D. grant (FPI-2005).

References

[1] E. Sahin, "Swarm robotics: From sources of inspiration to domains of application," in *Swarm Robotics Workshop: State-of-the-art Survey*, ser. Lecture Notes in Computer Science, E. Sahin and W. Spears, Eds., no. 3342. Berlin Heidelberg: Springer-Verlag, 2005, pp. 10–20.

[2] A. T. Hayes and P. Dormiani-Tabatabaei, "Self-organized flocking with agent failure: Off-line optimization and demonstration with real robots," in *IEEE International Conference on Robotics Automation*. Piscataway, NJ: IEEE Press, 2002, pp. 3900–3905.

[3] H. Tanner, A. Jadbabaie, and G. Pappas, "Stable flocking of mobile agents, part i: fixed topology," in *Proceedings. 42nd IEEE Conference on Decision and Control*, vol. 2, December 2003, pp. 2010–2015.

[4] I. Navarro, A. Gutiérrez, F. Matía, and F. Monasterio-Huelin, "An approach to flocking of robots using minimal local sensing and common orientation," in *Hybrid Artificial Intelligent Systems: 3th International Workshop, HAIS 2008*, ser. Lecture Notes in Artificial Intelligence (LNAI), E. Corchado, A. Abraham, and W. Pedrycz, Eds. Berlin, Germany: Springer-Verlag, 2008, vol. 5271, pp. 616–624.

[5] T. Balch and R. Arkin, "Behavior-based formation control for multi-robot teams," *IEEE Transactions on Robotics and Automation*, vol. 14, no. 6, pp. 926–939, 1998.

[6] W. Spears, D. Spears, J. Hamann, and R. Heil, "Distributed, physics-based control of swarms of vehicles," *Autonomous Robots*, vol. 17, pp. 137–162, 2004.

[7] I. Navarro, J. Pugh, A. Martinoli, and F. Matia, *Distributed Autonomous Robotic Systems 8*. Springer Berlin Heidelberg, 2009, ch. A Distributed Scalable Approach to Formation Control in Multi-robot Systems, pp. 203–214.

[8] F. Bonsignorio, J. Hallam, and A. P. del Pobil, "GEM SIG review guidelines," Euron GEM Sig, Tech. Rep. [Online]. Available: http://www.heronrobots.com/EuronGEMSig/

[9] A. Marjovi and L. Marques, *Advances in Mobile Robotics*. World Scientific, 2008, ch. A Scalable benchmark for motion control of mobile robots, pp. 146–153.

[10] F. E. Schneider, D. Wildermuth, and A. Kruling, "Discussion of exemplary metrics for multi-robot systems for formation navigation," *International Journal of Advanced Robotic Systems*, vol. 2, no. 4, pp. 345–353, December 2005.

[11] S. V. Viscido, J. K. Parrish, and D. Grnbaum, "The effect of population size and number of influential neighbors on the emergent properties of fish schools," *Ecological Modelling*, vol. 183, pp. 347–363, 2005.

[12] N. Michael, M. M. Zavlanos, V. Kumar, and G. J. Pappas, "Maintaining connectivity in mobile robot networks," in *ISER*, ser. Springer Tracts in Advanced Robotics, O. Khatib, V. Kumar, and G. J. Pappas, Eds., vol. 54. Springer, 2008, pp. 117–126.

[13] T. Lochmatter, P. Roduit, C. Cianci, N. Correll, J. Jacot, and A. Martinoli, "SwisTrack - a flexible open source tracking software for multi-agent systems," in *IEEE/RSJ 2008 International Conference on Intelligent Robots and Systems (IROS 2008)*. IEEE, 2008, pp. 4004–4010.

[14] J. Pugh, X. Raemy, C. Favre, R. Falconi, and A. Martinoli, "A fast on-board relative positioning module for multi-robot systems," *IEEE/ASME Transactions on Mechatronics, Focused Section on Mechatronics in Multi Robot Systems*, 2009.

[15] A. K. Das, R. Fierro, V. Kumar, B. Southall, J. Spletzer, and C. J. Taylor, "Real-time vision-based control of a nonholonomic mobile robot," in *In IEEE Conference on Robotics and Automation*, 2001, pp. 1714–1719.

[16] R. L. Graham, "An efficient algorithm for determining the convex hull of a finite planar set," *j-INFO-PROC-LETT*, vol. 1, no. 4, pp. 132–133, 1972.

[17] H. Edelsbrunner, D. Kirkpatrick, and R. Seidel, "On the shape of a set of points in the plane," *IEEE Transactions on Information Theory*, vol. 29, pp. 551–559, July 1983.

[18] H. Kremer and E. W. Weisstein, "Isoperimetric quotient," MathWorld–A Wolfram Web Resource, Tech. Rep. [Online]. Available: http://mathworld.wolfram.com/IsoperimetricQuotient.html

[19] A. Gutiérrez, A. Campo, M. Dorigo, J. Donate, F. Monasterio-Huelin, and L. Magdalena, "Open e-puck range and bearing miniaturized board for local communication in swarm robotics," in *Proceedings of the IEEE International Conference on Robotics and Automation*. Piscataway, NJ: IEEE Press, 2009, pp. 3111–3116.